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NEW SYSTEM FOR APPROACHING THE DESIGN OF SOUNDING ROCKETS

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ABSTRACT

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Description of a simple calculation method which enables a designer to pinpoint the principal characteristics of a rocket meeting the requirements laid down by the design problem for a single-stage rocket in vertical ascent. The method can be used also to compare the influence of some parameters on the characteristics and performance of a rocket.

AUTHOR

Introduction

In general, to establish a project for a vertical ascent, single-stage rocket, tests are usually made in advance. Such tests are based on similar rockets and on personal experience. A digital computer is used to assist the project work. /3*

This report is intended to provide designers with a simple method for calculating and pinpointing the main features of a rocket, and which satisfies the requirements laid down by the design problem. This method can also be used to compare the influence of some parameters on the characteristics and performance of rockets.

With successive approximations it is possible to satisfactorily approach acceptable results for setting up a project.

*Numbers given in margin indicate pagination in original foreign text.

The introduction of some incommensurable quantities permitted a selection to be made, because of the incommensurability of solutions, among similar rockets which might satisfy requirements.

The method has been imposed on analytical solutions which need only some simplification that can be eliminated with the iteration of the application of the method.

In order to draw up the incommensurable graphs herein, a digital computer was used, which permitted greater precision, while the generalization of the solution, on the contrary, remained valid.

1. Propelled Ascent /4

In order to calculate the velocity and the altitude attainable by a single-stage sounding rocket launched vertically, it is necessary to integrate the valid differential equation for the propelled portion of the rocket's flight

$$\frac{dv}{dt} = v_e \frac{G}{W} - g - \frac{Rg}{W} \qquad (1)$$

To simplify calculation, without detracting from the generality of the problem, the rocket was dealt with at zero angle of air trim on the trajectory, zero aerodynamic lift, and in a tranquil atmosphere.

The terms of the equation are explained as follows (1)

$$dv = v_e \frac{G}{W_0 - Gt} dt - g\,dt - \frac{C_R \frac{1}{2} \rho S_m v^2 g}{W_0 - Gt} dt \qquad (2)$$

It is not possible to integrate equation (2) analytically, because the coefficient of aerodynamic resistance Cr is a function of the Mach number of flight, the density of the air depends on the altitude, the effective velocity of the exhaust gas, on the ambient pressure (and therefore on the altitude), and the acceleration of gravity g depends on the altitude.

In order to integrate analytically, but approximately equation (2), some simplifying hypotheses are advanced:

$v_c = \overline{v}_c$ = constant (3): an error which can be made in calculating the thrust is 10 percent at the greatest. However, in an iteration of the calculation method, it is possible to use a more realistic average value.

$g = \overline{g}$ = constant (4): the error which is committed is negligible.

$\rho = \overline{\rho}$ = constant (5): an average value of the density of the propelled portion of the rocket's flight is assumed and this value is modified during iteration of the calculation.

$C_r = \overline{C}_r = f(V_r)$ (6): the velocity of sound is considered constant with the altitude, so that the coefficient of resistance depends only on the velocity. The error committed is not great inasmuch as the velocity of sound depends /5 on the square root of the absolute temperature $a = \sqrt{K\ RT}$.

$v = v_r$ (7): velocity v introduces the second member of equation (2), because the calculation of the resistance is at zero, according to the equation

$$dv_r = v_c \frac{G}{W_0 - Gt} dt - g\,dt \tag{8}$$

by which integrating the initial element t = 0 and t, the following is obtained

$$v_R = -\left[v_c \log\left(1 - \frac{Gt}{W_0}\right) + gt\right] \tag{9}$$

R = r = resistance

Velocity v_r is therefore greater than the effective velocity of flight at the same element t.

The resistance calculated with this v_r is greater than the effective resistance.

An error is therefore introduced into equation (2), which renders the velocity and the altitude attained by the rocket lower than in reality.

The following incommensurable variables are stated

$$\varepsilon = \frac{G t}{W_0} \qquad \varepsilon_c = \frac{G t_c}{W_0} \tag{10}$$

$$\tau = \frac{\bar{\rho} S_c v_e^2}{G/g \, v_e} \tag{11}$$

$$\sigma = \frac{G/g \, v_e}{W_0} \tag{12}$$

For a discussion and the significance of the incommensurable quantities, see paragraph 3.

Equation (9) is transformed /6

$$v_R = -v_e \left[\log (1-\varepsilon) + \frac{\varepsilon}{\sigma} \right] \tag{13}$$

Since the coefficient of resistance $\bar{C}_r = f\ (v_r)$, the following is obtained

$$\bar{C}_R = f(\varepsilon, \sigma) \tag{14}$$

By substituting positions (3), (4), (5), (7), (10), (11), (12), (14), in equation (2), we integrate between = 0 and = c = $\frac{G\ t_c}{W_0}$ to obtain burnout velocity v'_c.

This velocity will be lower than the real value since, in calculating the resistance, velocity v_r was taken greater than the real velocity. Such error was incurred by assuming as burnout velocity the average value

$$v_c = \frac{v_c' + v_R}{2} \tag{15}$$

between the velocity calculated with a resistance greater than the real resistance and the velocity calculated without resistance

$$V_c = V_e \left[-\log(1-\varepsilon_c) - \frac{\varepsilon_c}{\sigma} - \frac{T}{4} \int \text{[illegible]} \right] \tag{16}$$

Integrating again, we obtain the altitude attained by the rocket at burnout

$$z_c = \frac{V_e^2}{g} \left[\frac{1-\varepsilon_c}{\sigma} \log(1-\varepsilon_c) + \frac{\varepsilon_c}{\sigma} - \frac{\varepsilon_c^2}{2\sigma^2} - \frac{T}{4\sigma} \int_0^{\varepsilon_c} \int_0^{\varepsilon} \frac{\overline{C}_r \left[\log(1-\varepsilon) + \frac{\varepsilon}{\sigma} \right]^2}{1-\varepsilon} \, d\varepsilon \, d\varepsilon \right] \tag{17}$$

2. Inertial Ascent

After propellant decay, the rocket's vertical ascent continues due to inertia, according to the following differential equation /7

$$\frac{dv}{dt} = -g - \frac{R g}{W_c} = -g - \frac{C_r \frac{1}{2} \rho_t S_m v^2 g}{W_0 \left(1 - \frac{W_r}{W_0} \right)} \tag{18}$$

It is also impossible to integrate this equation analytically for the same reasons given as with equation (2).

The following simplifying hypotheses are advanced:

$g = \overline{g}$ = constant (4): the error committed is also negligible.

$\rho_t = \alpha \overline{\rho}$ = constant (19): a constant value of the density proportional to value $\overline{\rho}$ taken in the propelled portion of the rocket's flight is assumed. (It is possible to decrease the error with successive iterations of the calculation.)

$C_r = \overline{C}_{rc}$ = constant (20): a constant value of the coefficient of resistance is assumed since much of the inertial flight is at supersonic velocity.

The incommensurable variable is introduced

$$\beta^2 = \frac{2(1-\epsilon_c)}{\bar{C}_{Rc}\,\alpha\,\zeta\,\sigma} \tag{21}$$

Introducing the incommensurable variables (10), (11), (12), (21) and positions (4), (19), (20), equation (18) becomes

$$dv = -g\left[1+\frac{1}{\beta^2}\left(\frac{v}{\bar{v}_e}\right)^2\right]dt \tag{22}$$

Equation (22) is integrated with the following boundary conditions

$$\left.\begin{aligned} t &= t_c \\ v &= v_c \\ z &= z_c \end{aligned}\right\} \quad \text{initial} \tag{23}$$

$$v = 0 \quad \int \quad \text{final} \tag{24}$$

$$v = \bar{v}_e A\, \mathrm{tg}\left[(t_c - t)\frac{\bar{g}}{A\bar{v}_e} + \mathrm{arctg}\left(\frac{1}{A}\frac{v_c}{\bar{v}_e}\right)\right] \tag{25}$$

$$t_f - t_c = \frac{\bar{v}_e A}{\bar{g}}\,\mathrm{arctg}\left(\frac{1}{A}\frac{v_c}{\bar{v}_e}\right) \tag{26}$$

$$z_f - z_c = -\frac{\bar{v}_e^2}{\bar{g}}\beta^2 \log\cos\mathrm{arctg}\left(\frac{1}{A}\frac{v_c}{\bar{v}_e}\right) \tag{27}$$

3. Discussion and Significance of the Incommensurable Quantities

The selected incommensurable quantities have a physical significance /8

$\varepsilon_c = \frac{W_c}{W_0} = \frac{[illegible]}{W}$ (10) represents the percentage of propellant with respect to the total initial weight of the rocket.

$\zeta = \frac{S\,\bar{v}^2\,\bar{\rho}}{Q/2\;V}$ (11) represents the ratio "section:main-thrust".

$\sigma = \frac{[illegible]}{[illegible]}$ (12) represents the acceleration at launching.

For similar rockets, the ratio between the weight of the structure and the propeller, and the weight of the propellant may be kept constant. $\gamma = \frac{W_s}{W_p}$ (28) makes it possible to obtain the following definition of /9

$$\frac{W}{W_0} = 1 - \varepsilon_c (1 + \gamma) \qquad (29)$$

the ratio between the payload and the weight of the rocket at launching.

It is convenient, we note, to seek the minimum value of ϵ_c in order to have the maximum payload equal to launch weight.

$$\frac{1}{A^2} = \frac{\bar{c}_{Bc} \frac{1}{2} S_m \rho \bar{v}_c^2}{W_a + W_u} = \frac{R_f}{W_a} \qquad (21)$$

represents the ratio between the fictitious aerodynamic resistance of the rocket in inertial ascent calculated with velocity $v = \bar{v}_c$ and the rocket's weight at burnout. Thus the parameter $1/A^2$ represents the rocket's fictitious deceleration in inertial flight and therefore an estimate of aerodynamic breaking at thrust decay.

4. Method for Setting Up the Project

In order to establish a project for a single-stage sounding rocket which might fulfill our requirements and for determine its chief characteristics, it is necessary to specify in advance

- the aerodynamic shape and therefore the behavior of the coefficient of resistance (6), (20);
- the type of propellant and therefore v_c (3);
- the propeller system and the related load-bearing structure necessary; their weight can be considered proportional to the weight of the propellant $\gamma = \frac{W_s}{W_p}$; (28)

- the distribution of the density of the air and therefore $\bar{\rho}$ (5) and $\alpha\,\bar{\rho}$ (19);

- the average value of the acceleration of gravity $\bar{g}$ (4).

On the other hand, the following are known

$W_o = W_u + W_s + W_p$	total initial weight	(30)
$W_p = G\,t_c$	initial propellant weight	(31)
$S = \frac{G}{g}\,\overline{V_e}$	constant thrust	(32)

The variables in play are G, t_c, S, S_m, Wo, W_u, W_p, W_s, /10 while the relations which define the problem are seven in number. By using (28), (30), (31) and (32), it would be simple to reduce the problem to only four precise variables with three relations. All things considered, we would state that it is not suitable to proceed in this way. As a matter of fact, the variable that can be imposed on the problem from where all other quantities can be defined analytically, can also be one of the four variables specified by (28), (30), (31), (32). In this way, the designer has been afforded greater liberty in defining the rocket or rockets which can satisfy the requirements of the project itself.

By adopting appropriate criteria, suggested from time to time by the problem proposed, by means of equation (27) which connects the incommensurable quantities ϵ_c τ σ, it is possible to calculate the value by the quantities themselves (as shown in the graph of figure 5).

These four unknown variables, (connected by definitions (10), (11), (12)), of the incommensurable quantities ϵ_c τ σ, characterize a group of similar rockets. Since one of the four variables is established by the data of the problem, the remaining unknown quantities are measured. In this way the

assumed measured quantity permits selecting a specific rocket which meets the needs of the problem.

The exactness of the method depends on the values assigned to the constants and on the error introduced in the calculation of the aerodynamic resistance.

The constants $\overline{g}$ (4), $\overline{v}_c$ (3) can be substituted after the first calculation by more realistic values. The average value of the density $\overline{\rho}$ (5) in the rocket's propelled flight can be introduced in the expression τ (12) after altitude Z_c has been calculated. The uncertainty can therefore be reduced to a minimum amount.

The value of ratio α between the density of the air in the rocket's propelled flight and the density during the rocket's inertial flight can be substituted in the second approximation, as can be seen in paragraph 5, on the basis of the results obtained in the first calculation. /11

With regard to the error introduced by the vr and therefore the resistance, it is to be noted that it is possible to eliminate it completely by using a digital computer for integrating the incommensurable equations (2) and (18).

The graphs, like those referred to in paragraph 5, permit a general solution, dependent only on the coefficient of resistance Cr and the ratio α, because all other constants are introduced by the incommensurable quantities ϵ_c τ σ (10), (11) and (12).

The graphs were obtained by means of a digital computer, with the coefficient of resistance $\overline{C}r$ and $\overline{C}rc = 0.32$ and with $\alpha = 0.07$.

5. Numerical Application of the Method

Given the altitude to be attained Z_f and the weight of the payload W_u, the characteristics of the single-stage rocket of minimum initial weight W_o must be defined.

To find the solution, it is necessary, first of all, to specify the field of possible existence of the two incommensurable quantities τ and σ; this has become quite necessary, in connection with limiting the many solutions permitted by the ballistic treatment, in order to avoid unfeasible solutions.

Such limitations, well known to technicians and designers, are dictated by constructive practice and may therefore be modified on the basis of technological developments.

From the expressions

$$\frac{V_c}{V_L} = F'\left(\varepsilon_c, \tau, \sigma, \bar{C}_R\right) \tag{16}$$

$$\frac{Z_c}{V_c^2/g} = F''\left(\varepsilon_c, \tau, \sigma, \bar{C}_R\right) \tag{17}$$

$$\frac{Z_f}{V_c^2/g} = F'''\left(\varepsilon_c, \tau, \sigma, \bar{C}_R, \bar{C}_{Rc}, \alpha\right) \tag{27}$$

it is possible to represent graphically the maximum altitude Z_f attained by /12 the rocket as a function of the incommensurable quantity ϵ_c τ σ (as seen in figures 1, 2, 3 and 4).

Possible values of ϵ_c τ and σ are presented in figure 5. This is in connection with altitude Z_f, which is given as information for the problem.

The pair of values τ and σ are selected: they reduce ϵ_c in that for

$$\frac{W_u}{W_o} = 1 - \varepsilon_c\left(1+\gamma\right) \tag{35}$$

the minimum value of Wo has a minimum value of ϵ_c.

Knowing ϵ_c τ σ Wo W_u, it is possible to calculate with (28), (29), (31) and (32) all other quantities characteristic of the rocket in the first design approximation.

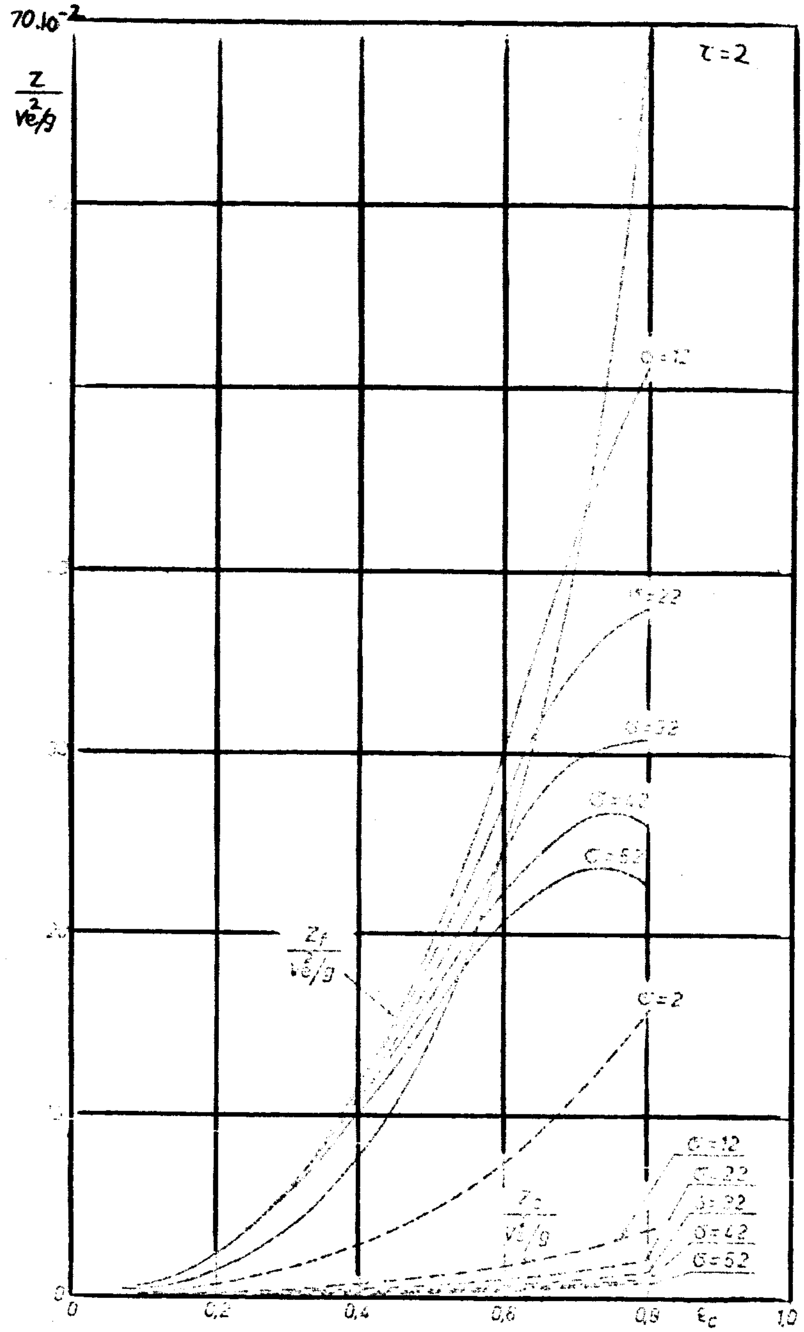

Figure 1

In the second approximation, it is possible to refine the values calculated, bearing in mind a more realistic distribution of the density of the air.

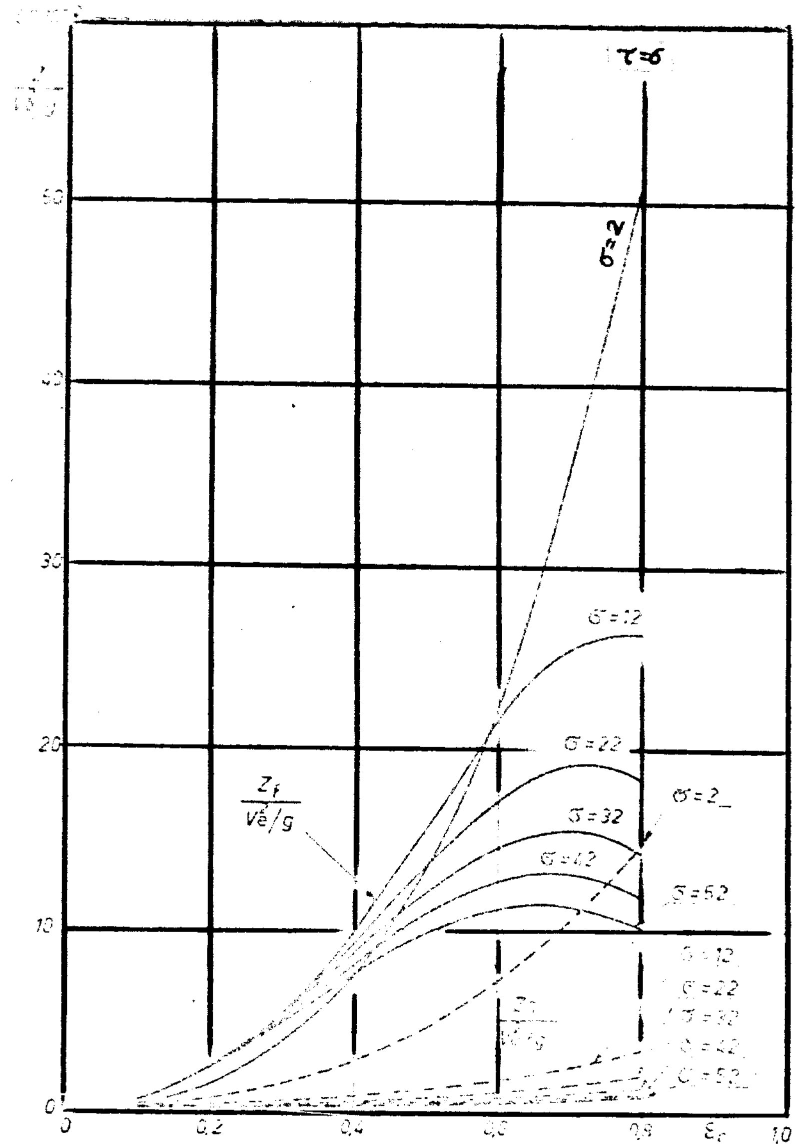

Figure 2

Z is calculated on the basis of (21) or on the graphs shown in figures 1, 2, 3 and 4. The average value of the density ρ at altitude Z_c (rocket in propelled flight) and at altitude $Z\rho$ (rocket in inertial flight) is determined. Hence, the following value can be determined $\alpha' = \frac{\rho}{\rho_f}$

On the basis of (20) or the graphs shown in figures 6, 7, 8 and 9, V/V_e is calculated.

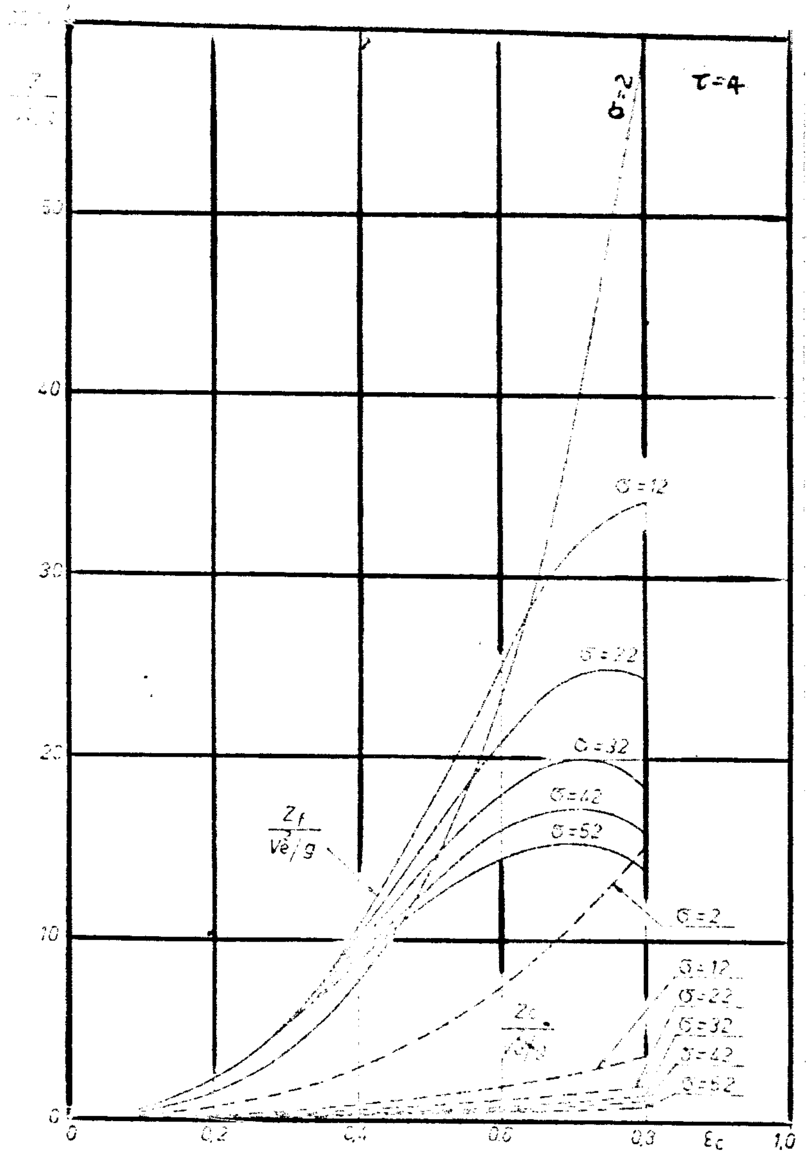

Figure 3

From (27) or the graphs shown in figure 10, we learn from $(\bar{z}_f - \bar{z}_c)/\bar{V}_e^2/g$ and $\bar{V}_c/\bar{V}_e$ the value of A.

In expression (21) of the incommensurable quantity, $A = \sqrt{\frac{2(1-\varepsilon_c)}{\tau_{ac}\,\alpha'\,\sigma}}$ /13

the new value calculated for α' is substituted. If τ remains the same, values must be found for ε_c and σ in order to satisfy (21) and (27) at the same time. The latter equation is shown in figure 5 a.

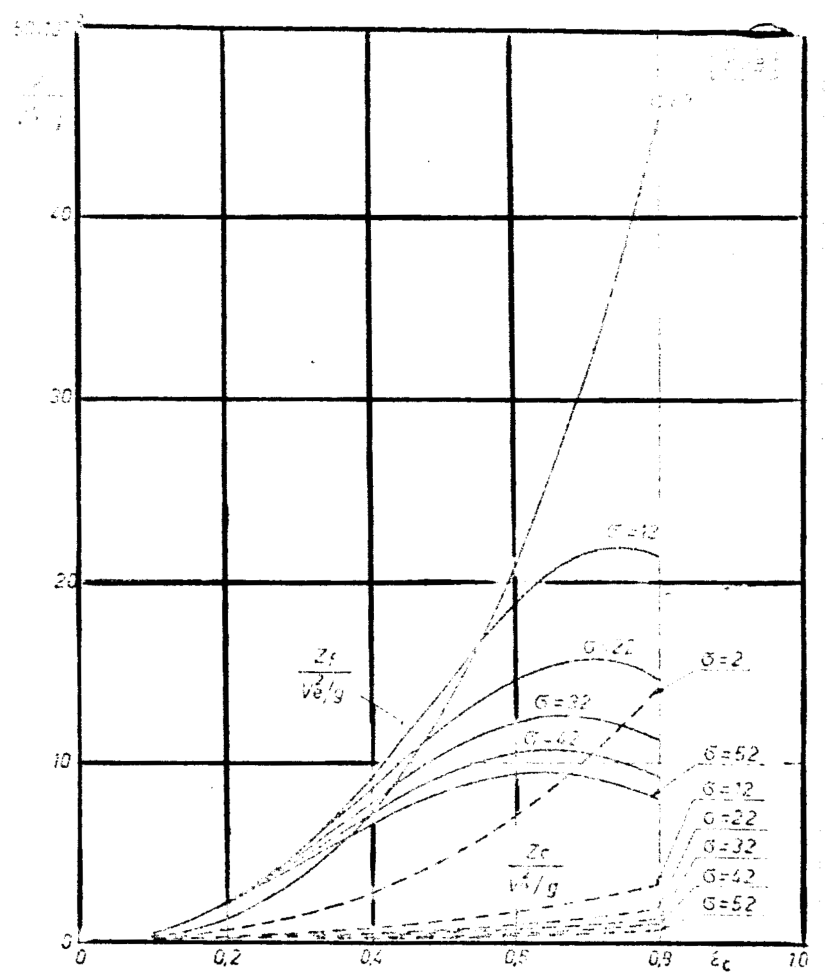

Figure 4

To find a pair of values (A, $\overline{C}_{rc}$, α', τ known), figure 5 shows the line which represents (21)

$$\sigma = \frac{2}{A^2\,\overline{C}_{Rc}\,\alpha'\,\tau} - \mathcal{E}_c\,\frac{2}{A^2\,\overline{C}_{Rc}\,\alpha'\,\tau} \qquad (33)$$

The point of intersection with the graph parameter τ represents the values of ϵ_c and σ.

In this way we may obtain a new rocket of more realistic performance and characteristics.

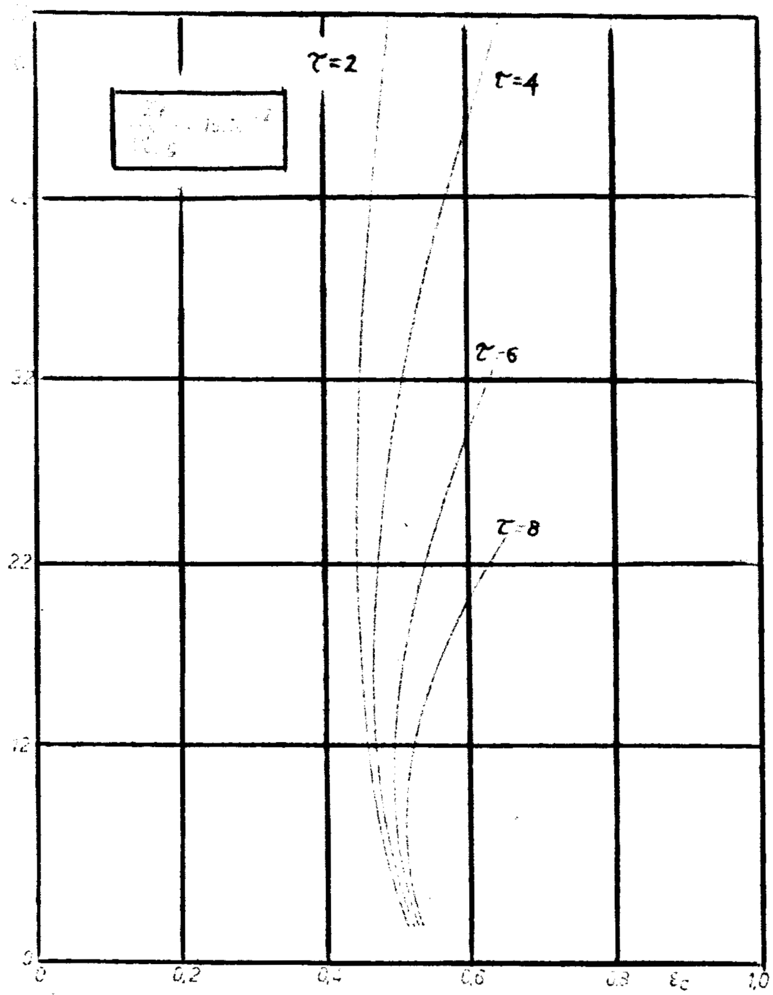

Figure 5

Numerical Example

A rocket of the following characteristics is taken

$$z_L = 160{,}000 \text{ m}$$

$$W_0 = 45 \text{ kg}$$

$$[illegible] = 1{,}930 \text{ m/sec}$$

$$\lambda = 0.40$$

It is assumed that g = g, Crc = 0.32 and α = 0.07. From the graphs in figures 1-10, we obtain for $\frac{z_L}{c^2/g}$ = 0.42 and the pair of values σ = 12 and τ = 2, which reduces ϵ_c = 0.685.

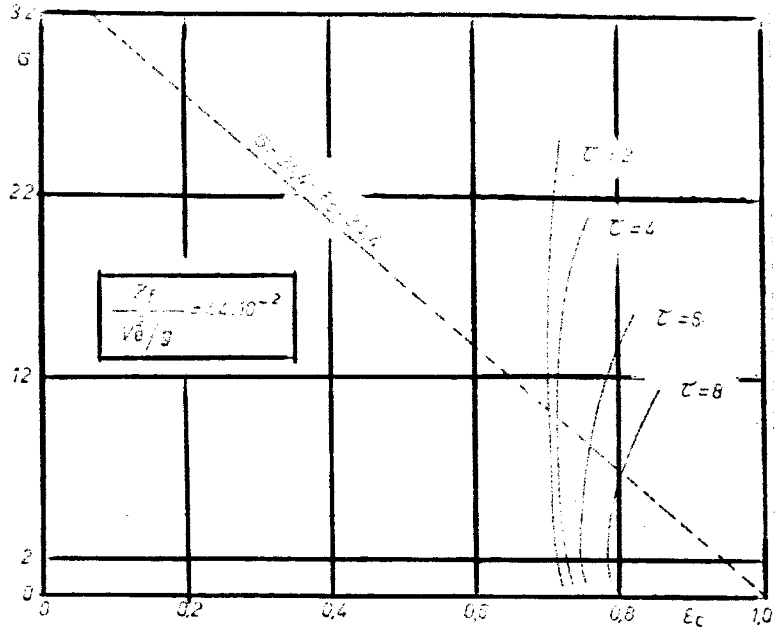

Figure 5 a

The following also is valid:

$$\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}} = 0{,}96, \quad \varepsilon_c/\bar{v}_e^2/g = 0.10, \quad \bar{\varrho} = 5.9 \cdot 10^{-2},$$

$$\varrho_i = 8.0 \cdot 10^{-4}, \quad \alpha' = 1.35 \cdot 10^{-4}, \quad A = 26.$$

Equation (21) therefore expresses /14

$$\sigma = 34.4 - \varepsilon_c \cdot 34.4$$

which is sketched in the graph of figure 5 a.

The point of intersection:

$$\sigma' = 10{,}2, \quad \tau' = 2, \quad \varepsilon_c' = 0{,}690,$$
$$Z_f' = 162.000 \text{ m}, \; Z_c' = 41.500 \text{ m}, \; v_c = 1870 \text{ m/sec}, \; W_o = 1300 \text{ kg},$$
$$W_p = 896 \text{ kg}, \; W_s = 359 \text{ kg}, \; t_c = 13{,}1 \text{ sec}, \; G = 68{,}5 \text{ kg/sec},$$
$$S = 13.300 \text{ kg}, \; S_m = 0.125 \text{ m}^2.$$

6. Numerical Application of the Method

Given the initial weight of rocket Wo, determine the possible number of rockets having the maximum possible altitude, Z_f and maximum payload W_u.

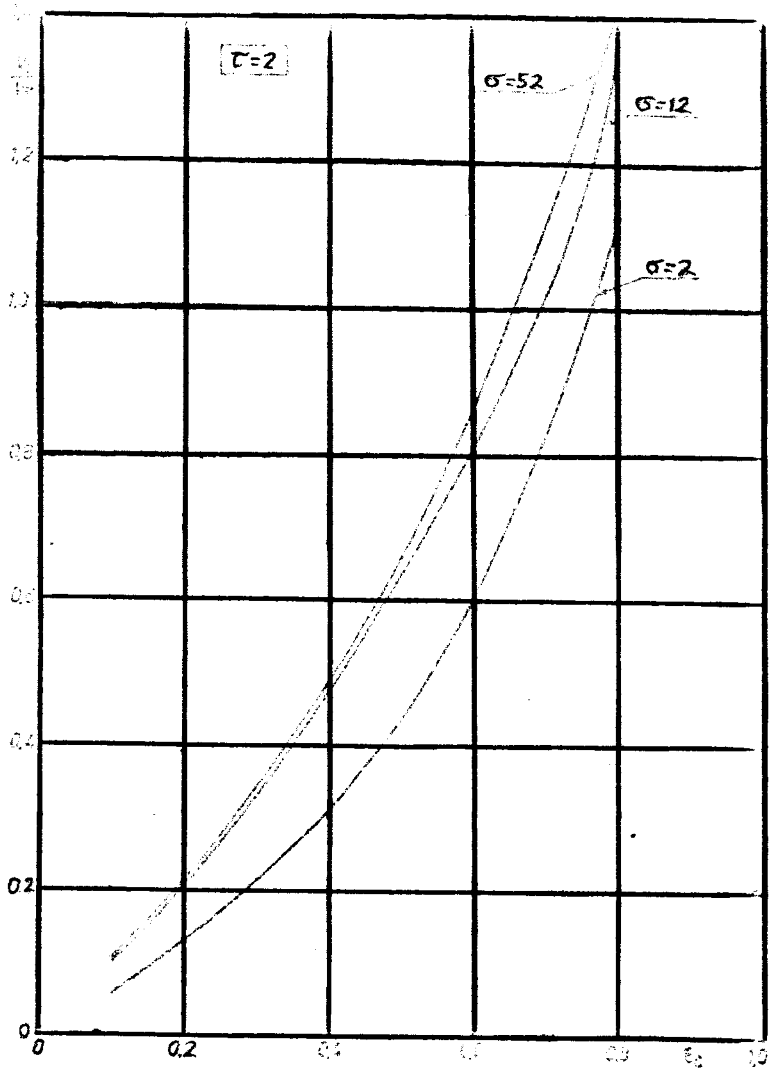

Figure 6

Assuming that the following values are known

$$\bar{V}_e, \bar{\sigma}, \bar{c}_k, \bar{c}_n, \alpha$$

with the help of (27), we trace the graphs ϵ_c = constant, which gives the altitude

$$z_f/\bar{V}_e^2/\bar{g}$$

as a function of σ and parameters for τ; see figures 11, 12, 13 and 14.

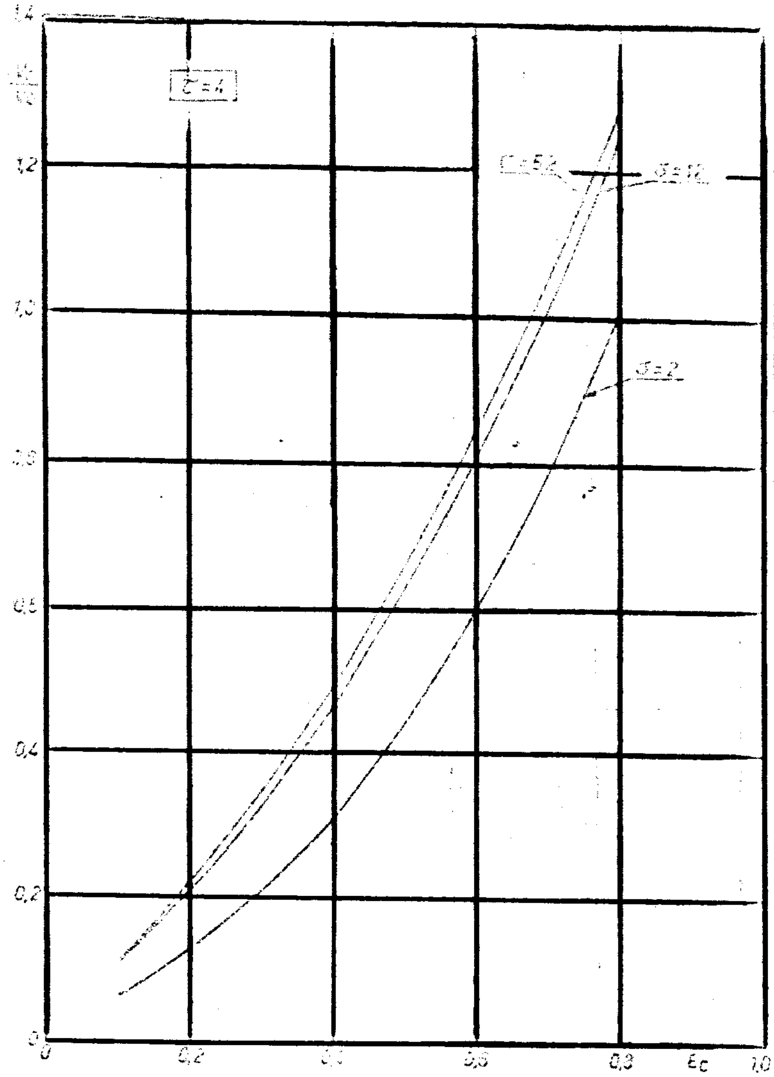

Figure 7

Payload W_u is established, and from (29) we obtain ϵ_c. From the corresponding graph we obtain, at the value ϵ_c, the maximum altitude attainable and the values of the two incommensurable parameters τ and σ.

These procedures are applied for the values of the payload selected and are then reported in the table as $Z_f = Z_f (W_u)$.

Numerical Example

The following values are given

$$W_o = 4000 \text{ kg}$$

$$v_c = 2500 \text{ m/sec}$$

$e = 0.29$
$C_{RC} = 0.32$
$a = 0.07$

$\frac{Z_f}{v_c^2/g}$	Z_f [m]	W_u [kg]	ε_c	τ	σ	W_p [kg]	W_s [kg]
0.472	295,000	390	[illegible]	2	6	[illegible]	810
0.392	[illegible]	648	0.650	2	7	[illegible]	752
0.354	222,000	775	0.625	2	7	2500	725
0.318	198,000	905	[illegible]	2	8	2400	695

Numerical Application of the Method

Given the specified single-stage rocket, determine the payload W_u which increases to the maximum the altitude Z_f attainable by the rocket. /15

From definitions (10) and (12) of the incommensurable quantities g_c and σ

$$\sigma = \frac{G\bar{v}_c}{\bar{g}W_o} = \frac{G\bar{v}_c}{\bar{g}W_p}\varepsilon_c = \sigma_1 \cdot \varepsilon_c \tag{34}$$

$$\sigma_1 = \frac{G\bar{v}_c}{\bar{g}W_p} = \text{constant} \tag{35}$$

Equation (34) is substituted into (27) and becomes a function of τ, ϵ_c and σ = constant.

By deriving and equating equation (27) to zero, it is possible to determine from the equation thus modified the value of ϵ_c which enables the rocket to attain the highest altitude. /16

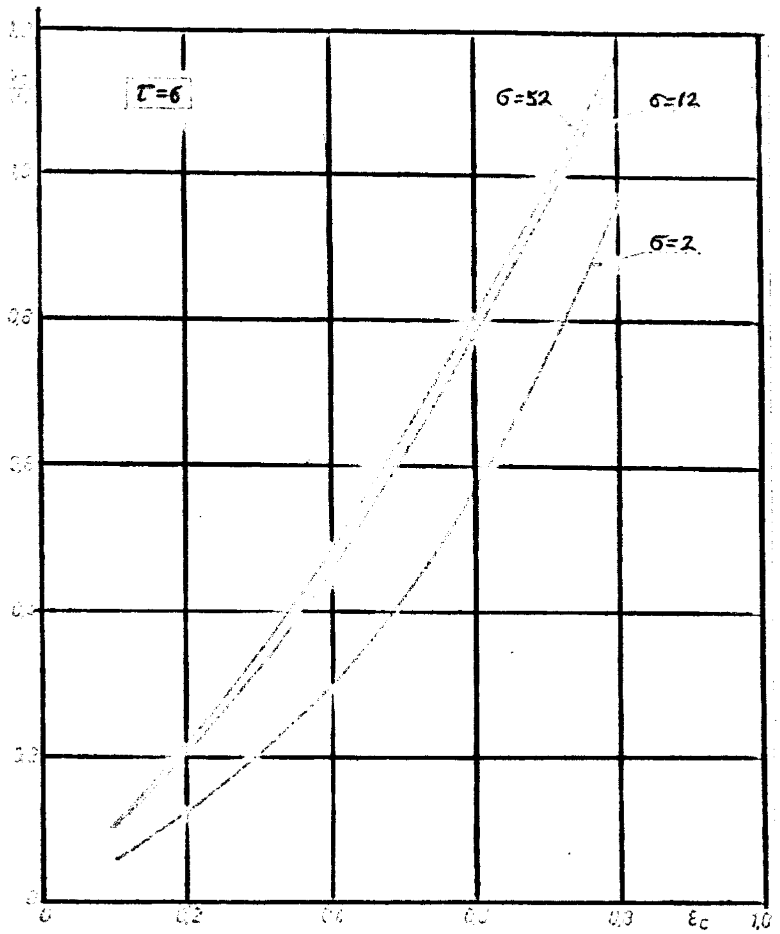

Figure 8

On the other hand, it is possible to obtain this value of ϵ_c from the graphs shown in figures 15, 16, 17 and 18, where according to the modified equation (27) Z_f is sketched as a function of σ_1 and τ.

Numerical Example

$$W_p = 34 \text{ kg}$$
$$Ws = 13.6 \text{ kg}$$
$$S_n = 0.0177 \text{ m}^2$$
$$G = 8.1 \text{ kg/sec}$$
$$t_c = 4.2 \text{ sec}$$
$$\bar{v}_e = 1860 \text{ m/sec}$$
[illegible]

$$\bar{\tau} = \frac{[illegible]}{G} = 4$$

$$\sigma_1 = \frac{G\,\bar{V}_e}{\bar{g}\,W_p} = 45$$

From figure 16, we obtain $\epsilon_c = 0.670$ from which

$$W_0 = 51.7 \text{ kg}$$
$$W_e = 4.1 \text{ kg}$$

From figure 2, we obtain $Z_f = 69{,}000$ m $Z_c = 3300$ m.

Appendix I

The increase in the altitude attainable by a rocket defined by factor /17
ϵ_c (which, as we know, is the ratio between the propellant weight and the initial

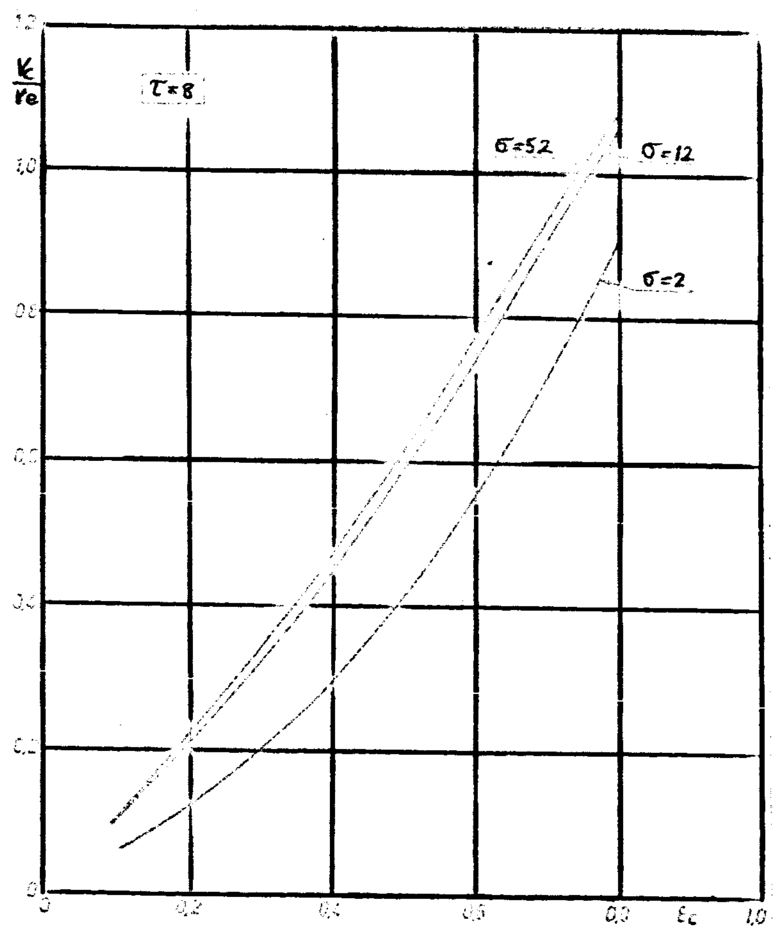

Figure 9

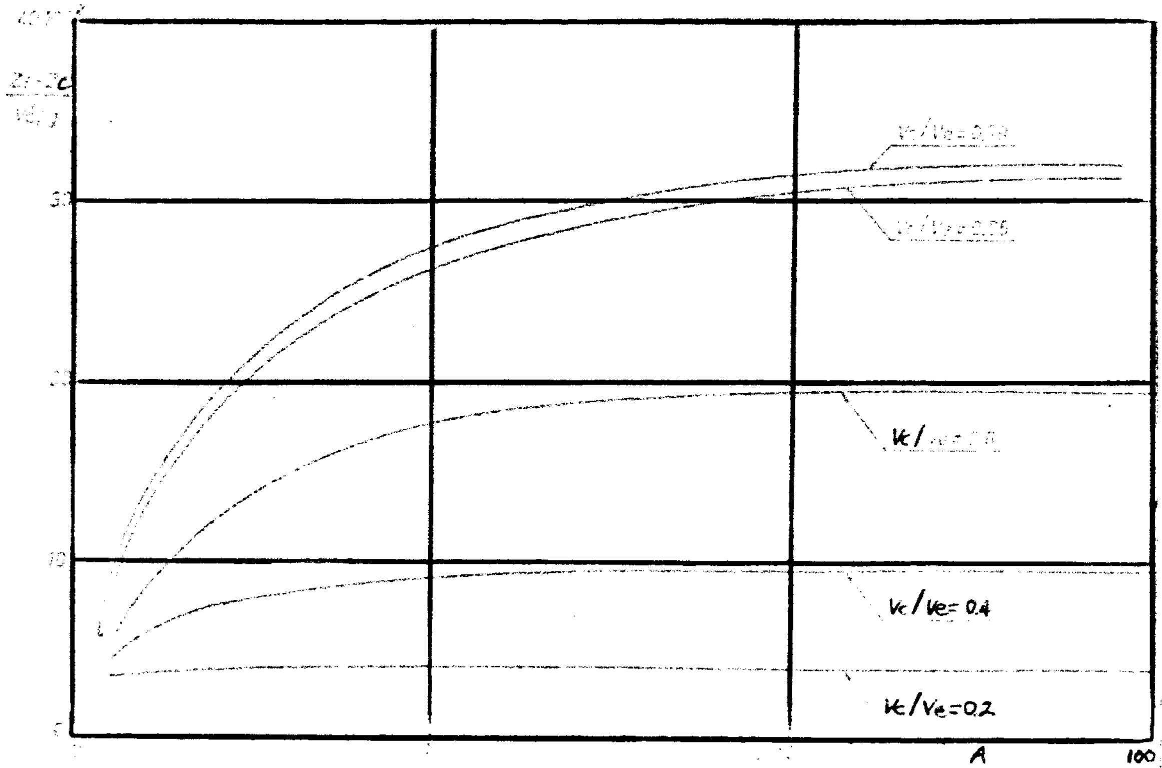

Figure 10

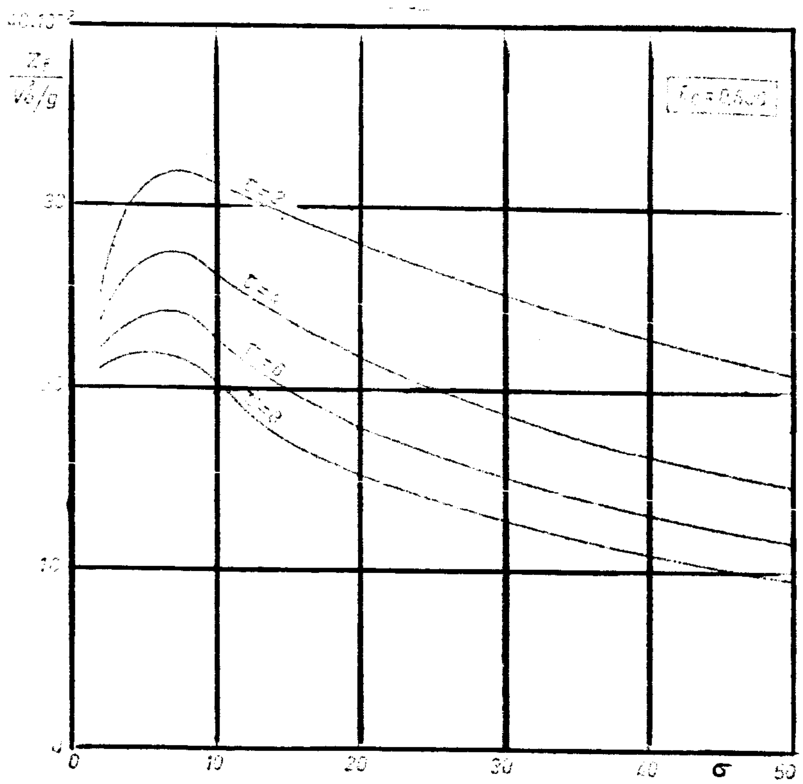

Figure 11

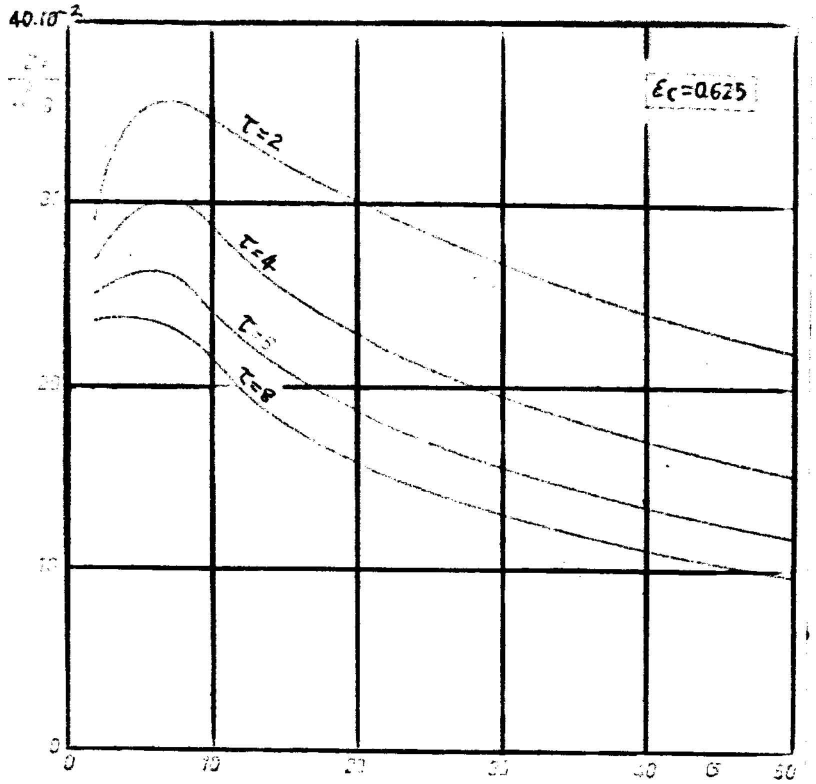

Figure 12

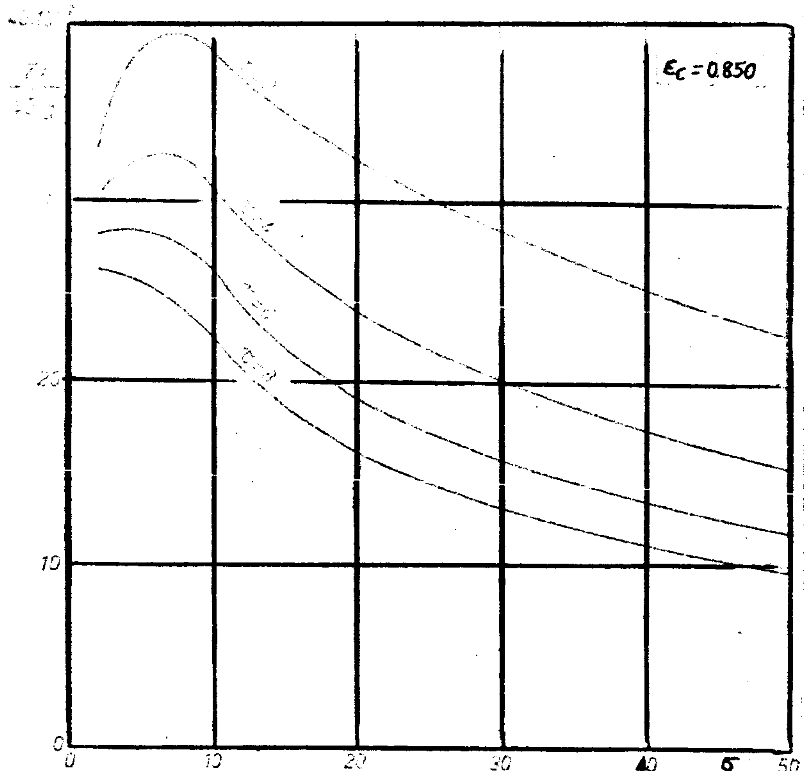

Figure 13

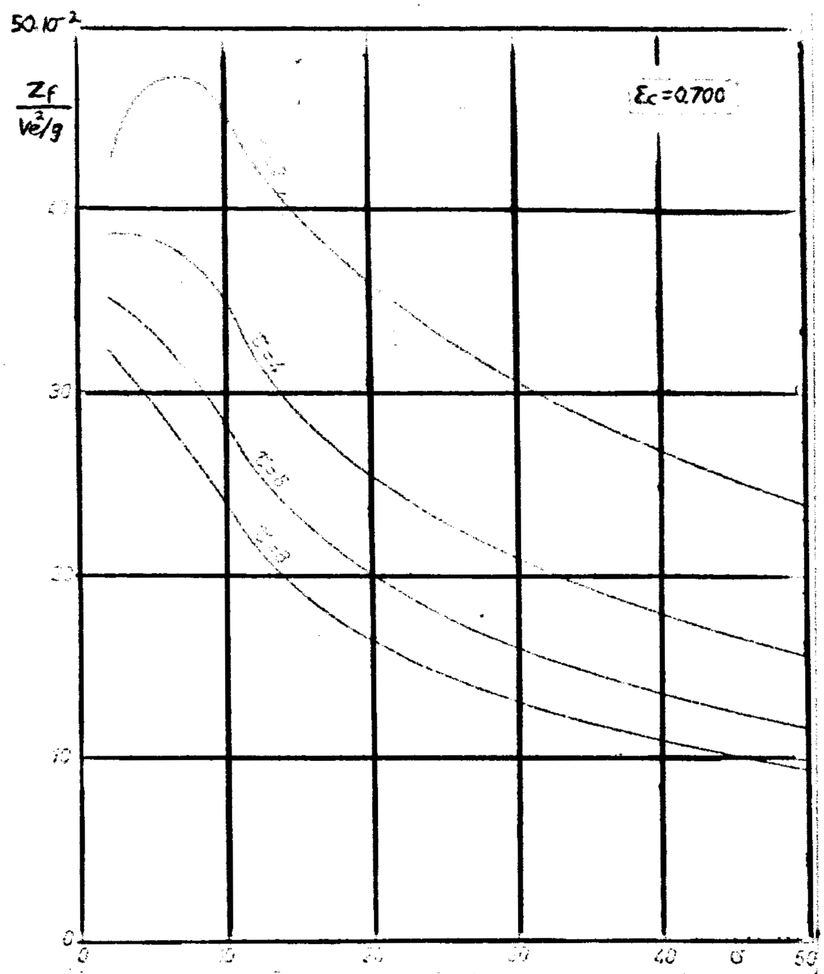

Figure 14

weight of the rocket) can be deduced easily by deriving equation (27) with regard to ϵ_c and equating it to zero.

$$\frac{d\left(z_f / \frac{v_e^2}{g}\right)}{d\epsilon_c} \; [illegible] \; \frac{d\left(z_\cdot / \frac{v_e^2}{g}\right)}{[illegible]} \; [illegible] \; (z_f - z_\cdot) \; [illegible] \; \partial(z_f - z_\cdot) \; d\left(\frac{v_c}{[illegible]}\right) \qquad (a)$$

The expression of single component terms in (a) are explained as follows:

$$[illegible] \frac{[illegible]}{v_e^2/g} \; [illegible] \; \frac{[illegible](1-\epsilon)}{[illegible]} - \frac{1}{\sigma} - \frac{\epsilon}{\sigma^2} - \frac{J}{4\sigma}\int_0^{\epsilon} \frac{\bar{c}_R\left[[illegible](1-\epsilon)[illegible]\right]}{1-\epsilon} \qquad (b)$$

$$\frac{\partial\left([illegible]\right)}{\partial\left([illegible]\right)} = [illegible] \; \frac{1}{\beta}\frac{v_c}{v_e} \; [illegible] \; \frac{\left(\frac{1}{\beta}\frac{v_c}{[illegible]}\right)^2}{[illegible]\left(\frac{1}{\beta}\frac{v_c}{[illegible]}\right)^2} \qquad (c)$$

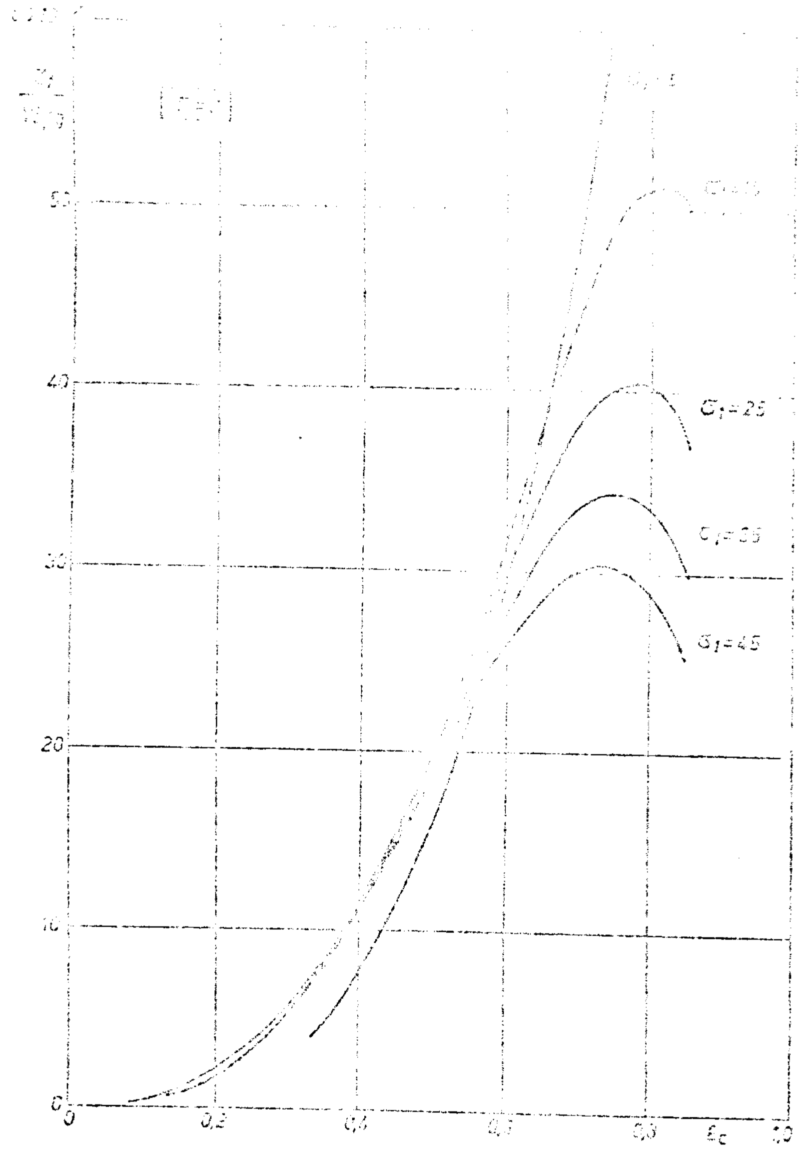

Figure 15

$$\frac{d(A^2)}{d\varepsilon_c} = -\frac{2}{\bar{C}_{Lc}\,\alpha J \sigma} \qquad \text{(d)}$$

$$\frac{\partial(z_f - z_c)}{\partial([illegible])} = \frac{\frac{1}{\sqrt{c}}}{1 + \left(\frac{1}{A}\right)[illegible]} \qquad \text{(e)}$$

$$\frac{d\left(\frac{V_E}{V_c}\right)}{d\varepsilon_c} = \frac{[illegible]}{1-\varepsilon_c}\,\frac{\varepsilon}{\sigma}\,\frac{J}{4}\,\frac{\bar{C}_R\left[\frac{\varepsilon_{cp}}{\sigma}\left(1-\varepsilon_c\right) + \frac{\varepsilon_c}{\sigma}\right]^2}{1-\varepsilon_c} \qquad \text{(f)}$$

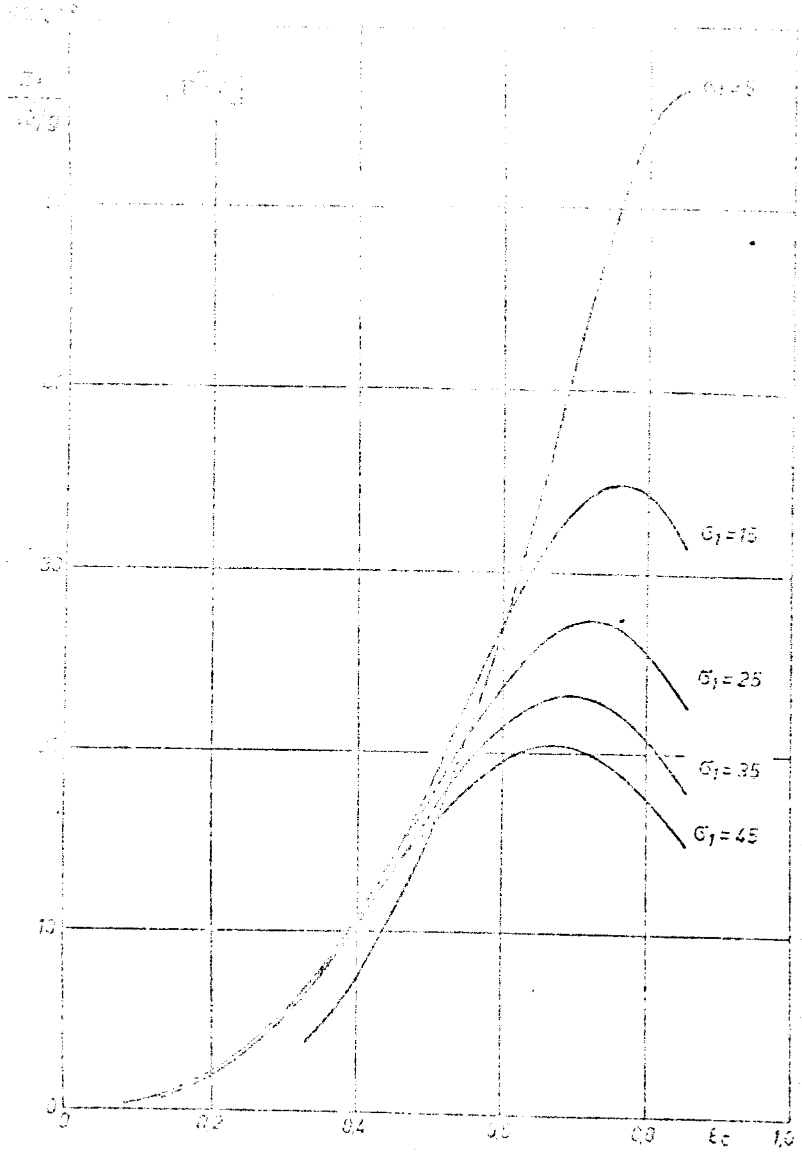

Figure 16

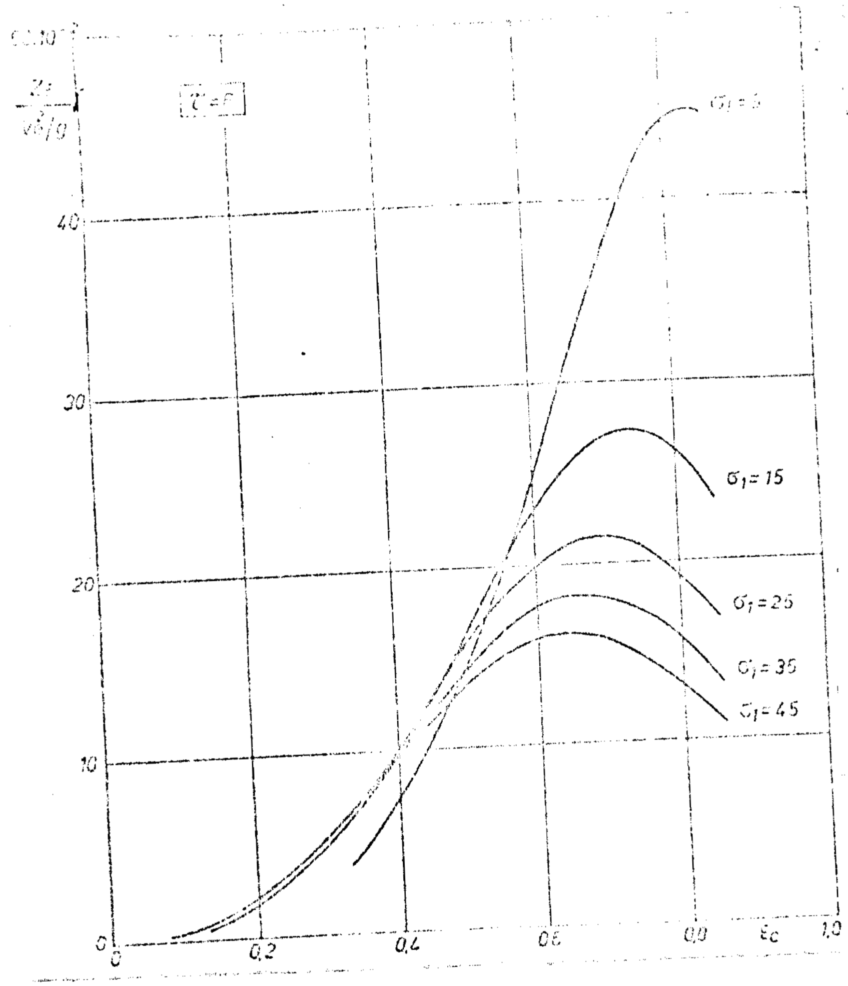

Figure 17

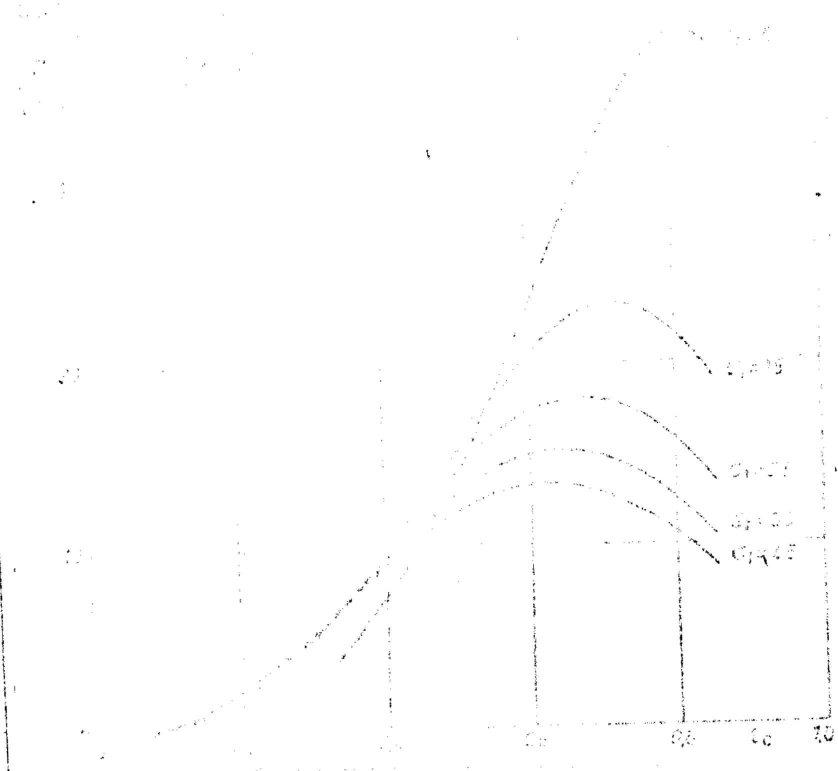

Figure 18

By substituting (b), (c), (d), (e) and (f) in (a), an equation is obtained in which τ and σ can be assumed as parameters and ϵ_c as an unknown quantity.

It is not possible to obtain the analytic expression of ϵ_c by the unlike pair τ and σ, since equation (a) is transcendental. However, it would be possible to show graphically the field of existence of τ and σ by which (a) can be solved and which are therefore values for which there is a maximum of the function $z_f/$[illegible]. This, however, would reduce the exactness of the solution of the problem and would be a useless complication to this method.

SYMBOLS

v	= rocket velocity	m/sec
v_c	= rocket velocity at burnout	m/sec
t	= time	sec
t_c	= time of combustion duration	sec
v_e	= effective velocity of exhaust gas	m/sec
W	= rocket weight	kg
Wo	= rocket weight at launching	kg
W_u	= weight of payload	kg
W_p	= propellant weight	kg
W_s	= structure weight	kg
W_c	= rocket weight at burnout	kg
G	= range weight of exhaust gas	kg/sec
R, r	= aerodynamic resistance of the rocket	kg
Cr	= f (M) = coefficient of aerodynamic resistance	
$\bar{Cr}$	= f (vr) = coefficient of resistance in propelled flight	
$\overline{Crc}$	= coefficient of resistance in inertial flight	

g	= acceleration of gravity	m/sec^2
S	= propellant decay	kg
S_m	= greater rocket section	m^2
ρ	= air density	$\frac{kg\ sec^2}{m^4}$
$\bar{\rho}$	= average density in propelled flight	$\frac{kg\ sec^2}{m^4}$
ρ_f	= average density in inertial flight	$\frac{kg\ sec^2}{m^4}$
α	= ratio of the density	
Z	= rocket altitude	m
Z_c	= altitude at burnout	m
Z_f	= highest altitude attained by the rocket	m
ϵ	= incommensurable quantity (10)	
ϵ_c	= incommensurable quantity calculated for $t = t_c$	
τ	= incommensurable quantity (11)	
σ	= incommensurable quantity (12)	
A^2	= incommensurable quantity (21)	

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